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Efficient Multi-unit Procurement Mechanism with Supply Disruption Risk

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Abstract
In this paper, we study the multi-attribute multi-unit procurement mechanism design problem facing a set of potential suppliers who suffer from disruption risks. Each supplier's production cost depends on its disruption probability, and both are private information. We propose a Vickery-Clark-Groves auction with disruption risk (VCG-DR) for this problem and show that the mechanism is incentive-compatible, individual-rational and social efficient. Moreover, we compare the performance of the proposed mechanism and the popular single-attribute multi-unit forward auction (SA-MFV) with reserved attribute by numerical experiments. The results show that VCG-DR outperforms SA-MFV in both *social efficiency* and *optimality*.

Keywords: multi-attribute reverse auction; supply disruption; mechanism design; VCG auction
Mathematics Subject Classification: *0*

1. Introduction

During the last decade, we witnessed many disruption events like earthquakes, hurricanes, strikes, machine breakdowns and man-made disasters in the world. These disruption events cause great loss to supply chains. The increasing disruption risk brings large difficulties for managers to choose suppliers when purchasing raw materials and components. In addition, the complexity of global supply chain makes it difficult for buyers to obtain information from their suppliers. The uncertainty of the supply also arises from the producing process itself in some industries, for example, the vaccine industry (Cho and Tang, 2013). Under this situation, how to choose efficient suppliers facing asymmetric information of supply uncertainty and production cost is an important problem faced by procurement managers.

On the other hand, reverse auction is widely adopted in procurement due to its excellent cost saving advantage comparing with other procurement methods (Pham et al., 2015). Many companies like HP, Dell and GE are using reverse auction to purchase their materials and services (Chen, 2014; Santamaría, 2015; Leu, 2008). However, the majority of auctions are price-only, which ignore other attributes other than the cost (Beil and Wein, 2003; Pham et al., 2015). This brings criticism to reverse auction since it may lead to troubles involving buyer-supplier trust and long-term relationship (Pham et al., 2015; Smeltzer & Carr, 2002). Besides, the reverse auction may cause greater loss compared with its cost saving when ignoring the non-price attributes such as quality, supply risk, etc. since the business is more and more relying on product quality and supply efficiency. Thus, the research on multi-attribute reverse auction is interesting and important (Pham et al., 2015).

In this work, we study the procurement mechanism design problem when the buyer's purchase quantity is multi-unit and the potential suppliers have uncertain supplies, and the order is dividable. We design a procurement auction to help the buyer to select suppliers and allocate the order efficiently among them. We first characterize the optimal allocation problem when all information are known. Then we propose a Vickrey-Clark-Groves type auction for this problem (we call it VCG-DR auction) and show that the mechanism is incentive compatible and individual rational. We generalize the concept of social efficiency of a mechanism to the case with supply disruption and show that the VCG-DR is efficient. Furthermore, we compared the performance of the VCG-DR auction and the popular single-attribute multi-unit forward Vickrey (SA-MFV) auction with reserved attribute by numerical experiments. The results show that, the VCG-DR is more efficient and optimal than the SA-MFV. Comparing with the existing literature, this work has the following contributions: (1) We first propose a multi-unit efficient procurement mechanism facing supply disruption; (2) We characterize the optimal allocation problem when all suppliers may suffer disruption events; (3) By numerical experiments, we also demonstrate that the proposed mechanism out-performs the popular SA-MFV with reserved attribute both in the social welfare and the buyer's expected profit.

The rest of the paper is organized as follows. In Section 2, we review some related literature to identify the research gap. We present the problem setting and analyze the symmetric case in Section 3. In Section 4, we propose the VCG-DR auction and study its properties. We compare the performance of the VCG-DR and the SA-MFV by numerical experiments in Section 5. The conclusions are given in Section 6.

2. Literature Review

In this paper, we study multi-unit multi-attribute efficient procurement mechanism design problem facing supply disruption. There are three streams of literature related to this study.

The first stream of literature related to this study is on supply chain risk management. In the last decades, since natural and man-made disasters are ubiquitous and supply chain risk management has attracted extensive interesting from academy and numerous literature have appeared. Fahimnia, et al (2015), Tang (2006), Tang and Musa (2011) gave extensive review. The majority of the literature study the managing methods and strategies facing unreliable suppliers, e.g. contract menu (Yang et al., 2009), business insurance (Dong and Tomlin, 2012) and dual sourcing (Huang and Xu, 2015; Tang, Gurnani and Gupta, 2014; Yang et al, 2012). There are some works studying supplier selection and order allocation problem when facing supply uncertainty (Federgruen and Yang, 2008; Sawik, 2014). The above works assume that the suppliers' disruption probabilities and production costs are complete information.

The second stream of literature related to this study is on multi-attribute reverse auction. Pham et al. (2015) gave an extensive review. Che (1993) first considered two-dimensional auctions and used a score function to aggregate price and quality. He analyzed the first-score, second score and second-preferred auction, and explored the revenue equivalence between them. Based on the work of Che (1993), a lot of researches have been conducted to explored the revenue equivalence between different auction formats (Branco, 1997; David et al., 2006). The above studies do not consider the attribute of supply disruption risk.

Chaturvedi and Martinez-de-Albeniz (2011) first studied the order allocation problem under the optimal mechanism in the presence of disruption risk. They didn't discuss the allocative efficiency of their mechanism. Similar to Chaturvedi and Martinez-de-Albeniz (2011), we also study the procurement mechanism design facing supply disruption risk, however, we focus on the efficient mechanism.

As to efficient multi-attribute auction design problem, Parkes and Kalagnanam (2005) proposed an efficient descending multi-attribute reverse auction, assuming that the buyer only buy one-unit product, the utility function is additive and the quality is discrete. They showed that the auction implements the one-side VCG (Vickrey-Clark-Groves) auction. Xiang et al. (2018) studied the efficient procurement mechanism design facing supply disruption risk. The above two studies considered only one-unit case. Xu and Huang (2017) studied efficient multi-unit multi-attribute procurement mechanism design problem. They proposed a primal-dual Vickrey (PDV) auction which implements the one-side VCG.

Our work is closely related to Xu and Huang (2017) and we study efficient multi-unit multi-attribute procurement mechanism design considering supply disruption. But there are two distinctions: (1). The non-price attribute considered in our work is disruption risk which makes the supplies uncertain. The PDV auction in Xu and Huang (2017) can't be used since it relies on the definition of over-demand set but this concept can't be defined in our problem. (2). In Xu and Huang (2017), the non-price attributes don't impact the quantity the buyers will get. So each supplier's contribution to the system depends only on its own information and can be calculated by dual theory. Here the non-price attribute is supply disruption probability, which affects the quantity the buyer can get in the long run. Therefore, the dual method used in Xu and Huang (2017) can't be used in our work.

3. The Problem Setting

Consider a buyer who faces Q unit demands, and wants to purchase products from a supplier pool (denoted by $\mathrm{N} =$ with $|\mathrm{N}| = n$). The suppliers may be disrupted during their production or delivery process. The buyer will get revenue r if it successfully sells one unit product. However, the buyer may get fewer products than the demand and this will incur shortage. Let the unit shortage cost be l. On the other hand, the buyer may also get more products than the demand and the leftover has zero salvage value. Suppose that the success probability attribute space A has finite valuations, i.e., $\mathrm{A} == \{a_1, a_2, \ldots, a_m\}$. With different level of success probability, the production cost of each supplier is different. For example, if supplier i produces with level j success probability, i.e., its success probability is $\rho_i = a_j$, the corresponding unit production cost of supplier i is $c_i = f_i(a_j)$. In the auction, suppliers are requested to submit their cost function f_i. Suppose that each supplier can only supply one unit. When a supplier has more than one unit, we can view the supplier as multi-agent supplier.

Suppose that the buyer will pay w_i to supplier i when it successfully delivers one-unit product to the buyer. Otherwise, if supplier i is selected as a winner and it suffers disruption, the buyer will charge supplier i pre-announced penalty p. Let q_{ij} be a binary variable which takes 1 if supplier i is a winner and the j th level of success probability is chosen (i.e., $\rho_i = a_j$), and 0 otherwise (we will discuss later how to determine the values of w_i and q_{ij}). Let Q^{real} be the number of the products that the buyer actually gets (which is a random variable). The buyer's expected payoff is

$$\begin{aligned} E[\Pi_0] &= E\left[r \times \min\left(Q^{real}, Q\right) - l \times \left(Q - Q^{real}\right)^+\right] - \sum_{i\in \mathrm{N}, j\in \mathrm{M}} q_{ij}\left(w_i - (1-\rho_i)p\right) \\ &= E\left[U(\boldsymbol{\rho},\mathbf{q})\right] - \sum_{i\in \mathrm{N}, j\in \mathrm{M}} q_{ij} w_i + \sum_{i\in \mathrm{N}, j\in \mathrm{M}} q_{ij}(1-\rho_i)p \end{aligned} \tag{0.1}$$

where $U(\boldsymbol{\rho},\mathbf{q}) = r \times \min\left(Q^{real}, Q\right) - l\left(Q - Q^{real}\right)^+$ is the revenue function, M denotes the index set of A, $\mathbf{q}$ is the allocation matrix (if $q_{ij} = 1$, it means supplier i is a winner and the j th level of success probability is chosen), and $\boldsymbol{\rho}$ is the corresponding success probability level vector (if $q_{ij} = 1$ then $\rho_i = a_j$, otherwise $\rho_i = 0$). When $\mathbf{q}$ is determined, $\boldsymbol{\rho}$ is also determined. When supplier i is the winner and the j th level of success probability is selected, its expected payoff is

$$E[\Pi_i] = q_{ij}\left(w_i - (1-\rho_i)p_i - f_i(\rho_i)\right) = q_{ij}\left(w_i - \left(1-a_j\right)p_i - f_i\left(a_j\right)\right). \tag{0.2}$$

In this paper, we focus on efficient mechanism, which maximizes the total profit of the buyer and the supplier(s). A social efficient mechanism can help the buyer and its suppliers to achieve better cooperation and future development (Parkes and Kalagnanam, 2005; Xu and Huang, 2017). When the suppliers' costs associating to each level of success probability are known to all, the social efficient allocation is the solution of the following optimization problem:

$$\begin{aligned} &\max_{\mathbf{q}} E\left[U(\boldsymbol{\rho},\mathbf{q}) - \sum_{i\in \mathrm{N}, j\in \mathrm{M}} q_{ij} c_i\right] \\ &s.t. \quad q_{ij} \in \{0,1\}, i \in \mathrm{N}, j \in \mathrm{M} \\ &\qquad \sum_{j\in \mathrm{M}} q_{ij} \le 1, i \in \mathrm{N} \end{aligned} \tag{0.3}$$

Note that, problem (0.3) is a stochastic programming since $U(\boldsymbol{\rho},\mathbf{q})$ involves random variable Q^{real}, and this makes the problem more complicated than the problem in Xu and Huang (2017), which is an 0-1 integer programming. Thus, methods like dual theory are not applicable to solve such problem.

Assumption 1. There are more than Q suppliers satisfying $\rho_i\left(r+l\right)-c_i>0$.

If otherwise, since $\rho_i\left(r+l\right)-c_i$ is the expected payoff brought by supplier i when the buyer faces shortage, it is easy to show that it is optimal to select all such suppliers as winners. To rule out trivial case, we make this assumption.

Let $\mathbf{q}$ be a feasible solution of (0.3), $\mathrm{I}_{\mathbf{q}}$ is the corresponding winner set and $\left|\mathrm{I}_{\mathbf{q}}=\right|=k$. The expected marginal profit corresponding to $\mathbf{q}$ generated by supplier i ($i\in\mathrm{I}_{\mathbf{q}}=$) is $MP_i=p\left\{Q_{-i}^{real}\le Q-1\middle|\mathbf{q}\right\}\rho_i\left(r+l\right)-c_i$, where Q_{-i}^{real} denotes the real quantity of the product received by the buyer from other suppliers excepted supplier i. When $k\le Q$, we have $MP_i=\rho_i\left(r+l\right)-c_i$. Otherwise, we have $MP_i<\rho_i\left(r+l\right)-c_i$.

For supplier j ($j\in\mathrm{N}\setminus\mathrm{I}_{\mathbf{q}}$), its expected marginal profit corresponding to $\mathbf{q}$ is $MP_j=p\left\{Q^{real}\le Q-1\middle|\mathbf{q}\right\}\rho_j\left(r+l\right)-c_j$, which is the profit difference of adding supplier j in $\mathrm{I}_{\mathbf{q}}$.

We further define the substitutional marginal profit corresponding to $\mathbf{q}$ of supplier j, $j\in\mathrm{N}\setminus\mathrm{I}_{\mathbf{q}}$ for supplier $i\in\mathrm{I}_{\mathbf{q}}$ as $SMP_{j,i}=p\left\{Q_{-i}^{real}\le Q-1\middle|\mathbf{q}\right\}\rho_j\left(r+l\right)-c_j-MP_i$, which is the profit change of removing supplier i from $\mathrm{I}_{\mathbf{q}}$ and adding supplier j in $\mathrm{I}_{\mathbf{q}}$.

Theorem 1. $\mathbf{q}^*$ *is an optimal solution of (0.3) if and only if* $MP_i\ge 0$ *for* $i\in\mathrm{I}_{\mathbf{q}^*}$, $MP_j\le 0$ *for* $i\in\mathrm{I}_{\mathbf{q}^*}$, *and* $SMP_{j,i}\le 0$ *for* $i\in\mathrm{I}_{\mathbf{q}^*}, j\in\mathrm{N}\setminus\mathrm{I}_{\mathbf{q}^*}$.

Proof. Without loss of generality, let supplier 1 to supplier k are selected as winners. Therefore, the probability function of Q^{real} is given as follow:

$$p\left(Q^{real}=0\middle|k\right)=p_k^0=\left(1-\rho_1\right)\left(1-\rho_2\right)\mathrm{L}\ \left(1-\rho_k\right)$$

$$p\left(Q^{real}=1\middle|k\right)=p_k^1=\rho_1\left(1-\rho_2\right)\mathrm{L}\ \left(1-\rho_k\right)+\left(1-\rho_1\right)\rho_2\mathrm{L}\ \left(1-\rho_k\right)+\mathrm{L}\ +\left(1-\rho_1\right)\left(1-\rho_2\right)\mathrm{L}\ \rho_k$$

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$$p\left(Q^{real}=k\middle|k\right)=p_k^k=\rho_1\rho_2\mathrm{L}\ \rho_k.$$

And the expected revenue function is,

$$\begin{aligned}E^k\left[U\left(\boldsymbol{\rho},\mathbf{q}\right)\right]=&\ r\left[\left(1p_k^1+2p_k^2+\mathrm{L}\ +Qp_k^Q\right)+Q\left(p_k^{Q+1}+p_k^{Q+2}+\mathrm{L}\ +p_k^k\right)\right]\\&-l\left(Qp_k^0+\left(Q-1\right)p_k^1+\mathrm{L}\ +1p_k^{Q-1}\right)\end{aligned}$$

Case 1, add one supplier, say supplier $k+1$

Then the probability function of Q^{real} is,

$$p\left(Q^{real}=0\middle|k+1\right)=p_{k+1}^{0}=p_{k}^{0}\left(1-\rho_{k+1}\right)$$
$$p\left(Q^{real}=1\middle|k+1\right)=p_{k+1}^{1}=p_{k}^{1}\left(1-\rho_{k+1}\right)+p_{k}^{0}\rho_{k+1}$$
$$\cdots$$
$$p\left(Q^{real}=k\middle|k+1\right)=p_{k+1}^{k}=p_{k}^{k}\left(1-\rho_{k+1}\right)+p_{k}^{k-1}\rho_{k+1}$$
$$p\left(Q^{real}=k+1\middle|k+1\right)=p_{k+1}^{k+1}=p_{k}^{k}\rho_{k+1}$$

Then the expected revenue function is,

$$\begin{aligned}E^{k+1}\left[U\left(\boldsymbol{\rho},\mathbf{q}\right)\right]=r&\left[\left(1p_{k+1}^{1}+2p_{k+1}^{2}+\cdots+Qp_{k+1}^{Q}\right)+Q\left(p_{k+1}^{Q+1}+p_{k+1}^{Q+2}+\cdots+p_{k+1}^{k}+p_{k+1}^{k+1}\right)\right]\\&-l\left(Qp_{k+1}^{0}+\left(Q-1\right)p_{k+1}^{1}+\cdots+1p_{k+1}^{Q-1}\right)\end{aligned}$$

By calculating the difference of the expected payoffs, we get:

$$\begin{aligned}\Delta=r&\begin{bmatrix}1\left(p_{k+1}^{1}-p_{k}^{1}\right)+2\left(p_{k+1}^{2}-p_{k}^{2}\right)+\cdots+Q\left(p_{k+1}^{Q}-p_{k}^{Q}\right)\\+Q\left(p_{k+1}^{Q+1}-p_{k}^{Q+1}+p_{k+1}^{Q+2}-p_{k}^{Q+2}+\cdots+p_{k+1}^{k}-p_{k}^{k}+p_{k+1}^{k+1}-0\right)\end{bmatrix}\\&-l\left(Q\left(p_{k+1}^{0}-p_{k}^{0}\right)+\left(Q-1\right)\left(p_{k+1}^{1}-p_{k}^{1}\right)+\cdots+1\left(p_{k+1}^{Q-1}-p_{k}^{Q-1}\right)\right)\end{aligned}$$

and we have

$$p_{k+1}^{0}-p_{k}^{0}=-p_{k}^{0}\rho_{k+1}$$
$$p_{k+1}^{1}-p_{k}^{1}=\left(p_{k}^{0}-p_{k}^{1}\right)\rho_{k+1}$$
$$\cdots$$
$$p_{k+1}^{k}-p_{k}^{k}=\left(p_{k}^{k-1}-p_{k}^{k}\right)\rho_{k+1}$$

Thus, the marginal profit can be re-written as

$$\Delta=\rho_{k+1}\left(r+l\right)\left(p_{k}^{0}+p_{k}^{1}+p_{k}^{2}+\cdots+p_{k}^{Q-1}\right)-c_{k+1}=\rho_{k+1}\left(r+l\right)p\left\{Q_{k}^{real}\le Q-1\right\}-c_{k+1}=MP_{k+1}.$$

When $MP_{k+1}\le 0$ means that add supplier $k+1$ is not profitable.

Case 2, remove one supplier, say supplier j. Similarly, we could obtain that,

$$\Delta=-\left(\rho_{j}\left(r+l\right)p\left\{Q_{-j}^{real}\le Q-1\right\}-c_{j}\right)=-MP_{j}.$$

Then it means it is not profitable to remove any winner.

Case 3, substitute one supplier, say supplier j by supplier $k+1$. Then the difference of profit between two cases is,

$$\begin{aligned}\Delta&=\left(\rho_{k+1}\left(r+l\right)p\left\{Q_{-j}^{real}\le Q-1\right\}-c_{k+1}\right)-\left(\rho_{j}\left(r+l\right)p\left\{Q_{-j}^{real}\le Q-1\right\}-c_{j}\right)\\&=\left(\rho_{k+1}-\rho_{j}\right)\left(r+l\right)p\left\{Q_{-j}^{real}\le Q-1\right\}-\left(c_{k+1}-c_{j}\right)\\&=\rho_{k+1}\left(r+l\right)p\left\{Q_{-j}^{real}\le Q-1\right\}-c_{k+1}-MP_{j}\end{aligned}$$

It means not profitable to replace any winner by non-winner. **Q.E.D.**

Since Q^{real} is the sum of n random variables, it is difficult to characterize the optimal solution. Note that $p\left\{Q_{-i}^{real} \le Q-1|\mathbf{q}\right\}$ and $p\left\{Q^{real} \le Q-1|\mathbf{q}\right\}$ depend on other winners' disruption probabilities, so one supplier's contribution to the system depends on other suppliers' disruption probabilities. A popular way to solve stochastic programming is to develop heuristic algorithm (Patro et al., 2018), and Theorem 1 allows us to construct such algorithm and accelerate it.

4. The VCG Auction with Disruption Risk

In this section, we propose a mechanism to induce suppliers to reveal their information truthfully. To this end, we propose a procurement mechanism following the idea of Vickery-Clark-Groves auction (VCG-DR auction). The sequence of the events is:

1. The buyer announces the auction rules:
   a) The suppliers should bid their cost functions;
   b) The order allocation rule is $\kappa:\left(f_1',\ldots,f_n'\right)\to\mathbf{q}$, where f_i' is supplier i's bid and $\mathbf{q}$ solves problem (0.3);
   c) The payment to winning supplier i is $\psi: w_i = f_i'(\rho_i)+\prod(\mathrm{N})-\prod(\mathrm{N}\setminus i)$;
   d) If a supplier is selected as a winner but it can't deliver product to the buyer, this supplier will pay p to the buyer;
2. Supplier i ($i\in\mathrm{N}$) submits its bid f_i' by sealed format to the buyer;
3. The buyer solves problem (0.3) and announces the winners. If $q_{ij}=1$, supplier i is a winner with j th level is selected. We assume no transaction between suppliers;
4. Each winner produces the product, and delivers the product to the buyer if no disruption happens. Otherwise, it has to pay p to the buyer.

Similar to VCG mechanism, a winner's payment in VCG-DR depends on the supplier's contribution. However, there are some distinctions between VCG-DR and traditional VCG mechanism. In VCG-DR, if a winner fails to deliver product to the buyer, it has to pay a penalty to the buyer. This case does not happen under traditional VCG mechanism. So our mechanism is a generalization of traditional VCG mechanism. Since we consider the case that the suppliers have private information on their costs and disruption probabilities, it is very difficult to compute a supplier's contribution.

Proposition 1 (Incentive Compatibility, IC). *In the VCG-DR, truthful bidding is a Bayesian Nash Equilibrium.*

Proof. Suppose that all suppliers except supplier i bid their true information. We shall study supplier i's bidding strategy. When supplier i bids $\left(c_i,\rho_i\right)$ truthfully, the corresponding allocation is $\mathbf{q}$ and the corresponding supplier pool is $\mathcal{N}$. The expected payoff of supplier i (with j th success probability level) is

$$\Pi_i = q_{ij}\left(\left(w_i - c_i\right) - \left(1-\rho_i\right)p\right).$$

If otherwise, it bids $\left(c_i', \rho_i\right)$, which is not its true information, the corresponding allocation is $\mathbf{q}'$ and the corresponding supplier pool is $\mathcal{N}'$. The expected payoff of supplier i (with j' th success probability level) is

$$\Pi_i' = q_{ij'}'\left(\left(w_i' - c_i\right) - \left(1-\rho_i\right)p\right).$$

We use contradiction to prove the theorem. If truth telling is not the dominate strategy for supplier i, there exists at least one bid $\left(c_i', \rho_i\right)$ that $\Pi_i' > \Pi_i$.

There are four possible cases: (a) $q_{ij} = 1$ and $q_{ij'}' = 1$; (b) $q_{ij} = 0$ and $q_{ij'}' = 1$; (c) $q_{ij} = 1$ and $q_{ij'}' = 0$; (d) $q_{ij} = 0$ and $q_{ij'}' = 0$. The last two cases are obviously none-profitable, thus we focus on the case (a) and (b).

Case (a) In this case, supplier i is the winner in both cases and we have

$$\begin{aligned}\Pi_i' - \Pi_i &= \left(w_i' - c_i\right) - \left(1-\rho_i\right)p - \left(w_i - c_i\right) + \left(1-\rho_i\right)p = w_i' - w_i \\ &= \Pi\left(\mathcal{N}'\right) - \Pi\left(\mathcal{N}'\backslash i\right) + c_i' - \Pi\left(\mathcal{N}\right) - \Pi\left(\mathcal{N}\backslash i\right) - c_i\end{aligned}$$

Note that $\Pi\left(\mathcal{N}'\backslash i\right) = \Pi\left(\mathcal{N}\backslash i\right)$, then we have

$$\Pi_i' - \Pi_i = \Pi\left(\mathcal{N}'\right) + c_i' - c_i - \Pi\left(\mathcal{N}\right)$$

$$= U\left(\boldsymbol{\rho},\mathbf{q}'\right) - \sum_{t\in\mathcal{N}\backslash i, s\in\mathcal{M}} q_{ts}'c_t - q_{ij'}'c_i' + c_i' - c_i - \Pi\left(\mathcal{N}\right) = U\left(\boldsymbol{\rho},\mathbf{q}'\right) - \left(c_i + \sum_{t\in\mathcal{N}\backslash i, s\in\mathcal{M}} q_{ts}'c_t\right) - \Pi\left(\mathcal{N}\right) > 0$$

This contradicts the fact that $\mathbf{q}$ maximize the social profit.

Case (b) In this case, supplier i is the not a winner when bid truthfully and it is a winner when cheating. Then we have

$$\begin{aligned}\Pi_i' - \Pi_i &= \left(w_i' - c_i\right) - \left(1-\rho_i\right)p - 0 = \Pi\left(\mathcal{N}'\right) - \Pi\left(\mathcal{N}'\backslash i\right) + c_i' - c_i - \left(1-\rho_i\right)p \\ &= U\left(\boldsymbol{\rho},\mathbf{q}'\right) - \sum_{t\in\mathcal{N}\backslash i, s\in\mathcal{M}} q_{ts}'c_t - q_{ij'}'c_i' + c_i' - c_i - \left(1-\rho_i\right)p - \Pi\left(\mathcal{N}'\backslash i\right) \\ &= U\left(\boldsymbol{\rho},\mathbf{q}'\right) - \left(c_i + \sum_{t\in\mathcal{N}\backslash i, s\in\mathcal{M}} q_{ts}'c_t\right) - \left(1-\rho_i\right)p - \Pi\left(\mathcal{N}'\backslash i\right) > 0\end{aligned}$$

Note that supplier i is the not a winner when bid truthfully, which means that $\Pi\left(\mathcal{N}\right) = \Pi\left(\mathcal{N}\backslash i\right) = \Pi\left(\mathcal{N}'\backslash i\right)$, then we have

$$U\left(\boldsymbol{\rho},\mathbf{q}'\right) - \left(c_i + \sum_{t\in\mathcal{N}\backslash i, s\in\mathcal{M}} q_{ts}'c_t\right) > \left(1-\rho_i\right)p + \Pi\left(\mathcal{N}\right) > \Pi\left(\mathcal{N}\right)$$

This also contradicts the fact that $\mathbf{q}$ maximize the social profit when all supplier bid truthfully. Thus the theorem is proved. **Q.E.D.**

Proposition 1 indicates that the dominate strategy for suppliers is to bid their information truthfully in the VCG-DR. Similar to Xu and Huang (2017) and Parkes and Kalagnanam (2005), we extended the incentive compatibility to multi-attribute case.

Proposition 2 (Individual rationality, IR). *The VCG-DR mechanism is individual rational when p is sufficient small. That is, the suppliers can get non-negative profits by participating in the auction.*

Proof. First, when supplier i is not a winner, it gets zero profit. When supplier i is selected as a winner, its payoff is $\Pi_i = q_i w_i - q_i(1-\rho_i)p - q_i c_i$, by substituting the VCG-DR payment w_i, we get $\Pi_i = q_i\left[\prod(\mathrm{N}) - \prod(\mathrm{N} \setminus i) - (1-\rho_i)p\right]$, which is non-negative when p is sufficient small. Thus suppliers get non-negative profits by participating in the VCG-DR. **Q.E.D.**

Note that, the penalty is transaction payment between the buyer and suppliers, which doesn't affect the incentive compatibility of the VCG-DR. Thus, the buyer could encourage suppliers to participate in the auction by setting $p = 0$. But there is risk that suppliers may cheat by reporting that the disruption happens without executing the contract. This won't be a problem in some industries like the vaccine industry, since it is easy for the buyer to find out whether the suppliers have executed the contract or not. Another solution is to use asymmetric personalized penalty, for example, let the penalty to be certain ratio of the wholesale price that the buyer pays to the supplier.

Proposition 3 (Efficiency). *The VCG-DR is social efficient.*

Proof. Proposition 1 shows that all suppliers report their information truthfully, then the problem reduces to the case when all information are known to all. In the VCG-DR, the allocation rule is the solution of problem (0.3), which guarantees that the social welfare is the greatest. Thus, the proposition is proved. **Q.E.D.**

The proposition indicates that the VCG-DR is social efficient, i.e., maximizing the total profit of the buyer and supplier. Different from Parkes and Kalagnanam (2005) and Xu and Huang (2017), since the suppliers are not reliable, the buyer may get more or less than it order in the auction, which makes the problem more complicated and the results of these two works can not be applied to our problem.

5. Performance Analysis

In this section, we compare performance of the VCG-DR with that of the widely used, Single-attribute Multi-unit Forward Vickrey with Reserved Attribute (SA-MFV)

(Xu & Huang, 2017). In SA-MFV, the buyer (or auctioneer) sets a reserved level for non-price attributes, and bidders (suppliers) compete on the wholesale price of the product and the wholesale price is the highest losing bid. Note that, when all suppliers are reliable and the buyer sets a reserved success probability as $\rho_r = 1$, the SA-MFV with reserved attribute reduces to the SA-MFV defined in Xu & Huang (2017).

Lemma 1. *For a given reserved success probability ρ_r, the equilibrium bidding strategy of supplier i in SA-MFV is*

$$\beta_i(f_i, \rho_r) = f_i(\rho_r).$$

Proof. Suppose that supplier i bids $f_i'(\rho_r)$ and the lowest losing bid is y. If $f_i'(\rho_r) < f_i(\rho_r)$, there is a positive probability that it generates negative profit (when $f_i'(\rho_r) < y < f_i(\rho_r)$, it wins and the profit is $y - f_i(\rho_r) < 0$), and it's not profitable to bid less than $f_i(\rho_r)$. On the other hand, when supplier i bids greater than $f_i(\rho_r)$, its winning probability will decrease while its profit is unchanged (the payment is the lowest losing bid y which is independent to the supplier i's bid) when it wins. Thus, it's neither profitable to bid greater than $f_i(\rho_r)$. The lemma is proved. **Q.E.D.**

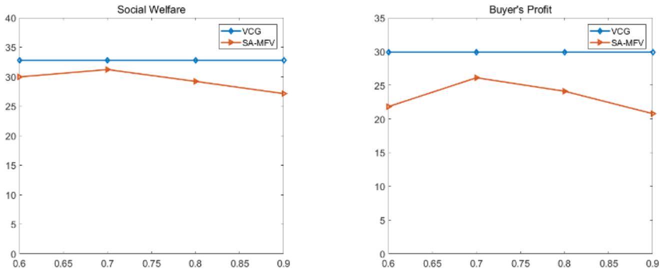
As a results, the real product received by the buyer Q^{real} follows the binomial distribution with parameter k and ρ_r, where k is the number of winners. And the probability function of Q^{real} is

$$p\left(Q^{real} = i \middle| k, \rho_r\right) = \binom{k}{i} (\rho_r)^i (1 - \rho_r)^{k-i}. \qquad (5.1)$$

Since all suppliers have the same success probability, the problem (0.3) is much easier to solve in this case. We need only rank all suppliers from the lowest to the highest by their costs they report. Then select the suppliers with lowest costs as winners sequentially, until all non-winners' marginal expected profits are negative.

Suppose that there are 10 potential suppliers and each supplier has 4 possible risk levels. The total demand faced by the buyer, $Q = 4$, the unit revenue for the buyer, $r = 20$, and the unit shortage loss, $l = 5$. The penalty that a winner has to pay if it fails to deliver the product, $p = 0.9$. For SA-MFV, we consider 4 reserved success probabilities, i.e., $\rho_r = 0.6, 0.7, 0.8, 0.9$.

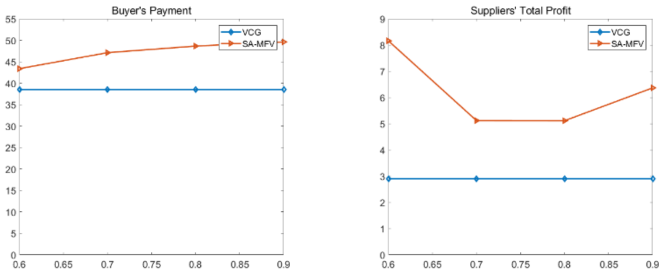
Figure 1 illustrates the total social welfare and the buyer's profit incurred by VCG-DR and SA-MFV with reserved attribute.



(a) (b)

Figure 1. The social profit and buyer's profit

From Figure 1, we find that VCG-DR outperforms SA-MFV with respect to efficiency and optimality. Also, we find that both the social profit and the buyer's profit first increases and then decreases in reserved success probability. This means that with proper reserved success probability, the social welfare can be maximized. Too low or too high reliability requirement may hurt the social profit and the buyer's profit.



(a) (b)

Figure 2. The buyer's payment and suppliers' total profit

Also, from Figure 2, we find that in SA-MFV, the buyer's payment increases in reserved success probability. This means that if a buyer wants a high reliable supply, it should pay more. We also find that suppliers' profit decreases first and then increases with respect to the reserved probability. Combining the results in Figure 1, we could conclude that a medium reserved success probability could increase the social welfare. However, the buyer exhausts the surplus and suppliers almost break even.

6. Conclusion

In this paper, we considered a procurement problem when the suppliers may suffer disruption events. We proposed the Vickery-Clark-Groves auction with disruption risk (VCG-DR) to help the buyer to select suppliers and extended the procurement problem to the multi-unit multi-attribute case. We demonstrated that the proposed mechanism achieves social efficiency and it is incentive compatible and individual rational. We compared the performance of the VCG-DR and the SA-MFV with reserved attribute by numerical experiments. The results show that the VCG-DR out-performs the SA-MFV with reserved attribute both in efficiency and optimality.

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